Math 151-L **Practice Problem Set 1** January 27, 2008

(1) Let $g(x) = \pi - x^2$ and $h(x) = cos(x)$. Find:

(a) $g(h(0))$

(b) $h(g(\sqrt{\frac{\pi}{2}})$

(c) $g(g(1))$

(d) $h(h(\frac{\pi}{2}))$

(2) Let $f(x) = \sqrt{x}$ and $g(x) = x^3 + 1$. Find:

(a) $f(g(2))$

(b) $g(f(4))$

(c) $f(f(16))$

(d) $g(g(0))$

(3) Let $g(x) = \sqrt{x}$. Find:

(a) $g(5s + 20)$

(b) $g(\sqrt{x} + 2)$

(c) $3g(5x)$

(d) $\frac{1}{g(x)}$

(e) $g(g(x))$

(f) $(g(x))^2 - g(x^2)$

(g) $g(\frac{1}{x})$

(h) $g((x-1)^2)$

(4) Express the following functions as peicewise functions:

(a) $f(x) = |x| + 3x + 1$

(b) $g(x) = |x| + |x - 1|$

(c) $h(x) = 3 + |2x - 5|$

(d) $I(x) = 3|x - 2| - |x + 1|$

(5) Find the domain of the following functions:

(a) $f(x) = \dfrac{x^2 + x - 5}{x^2 + 6x + 9}$

(b) $g(x) = \dfrac{\sqrt{\ln 6x - 3}}{x^3 - 2x^2 + x - 2}$

⋆⋆**HINT**: $x = 1$ is a root of the bottom function⋆⋆

(6) Let $f(x) = \dfrac{x^4 - 1}{x - 1}$

(a) Find $\lim_{x \to 1^+} f(x)$

(b) Find $\lim_{x \to 1^-} f(x)$

(c) Does $\lim_{x \to 1} f(x)$ exist?

(d) Find $\lim_{x \to 2^+} f(x)$

(e) Find $\lim_{x \to 2^-} f(x)$

(f) Does $\lim_{x \to 2} f(x)$ exist?

(7) Use the rules of limits (i.e. *the limit of a sum is the sum of the limits*) to do the following. **SHOW EVERY STEP**. Given that $\lim_{x \to a} f(x) = 2$, $\lim_{x \to a} g(x) = -4$, and $\lim_{x \to a} h(x) = 0$, Find the following limits.

(a) $\lim_{x \to a} [f(x) + 2g(x)]$

(b) $\lim_{x \to a} [h(x) - 3g(x) + 1]$

(c) $\lim_{x \to a} \dfrac{2}{g(x)}$

(d) $\lim_{x \to a} \dfrac{7g(x)}{2f(x) + g(x)}$

(e) $\lim_{x \to a} [g(x)]^2$

(8) Find the following limits (if they exist). If they do not exist, explain why.

(a) $\lim_{y \to 4} \frac{4 - y}{2 - \sqrt{y}}$

(b) $\lim_{x \to 9} \frac{x - 9}{\sqrt{x} - 3}$

(c) $\lim_{x \to \infty} \frac{\sqrt{3x^4 + x}}{x^2 - 8}$

(d) $\lim_{x \to 0} \frac{\sqrt{x - 4} - 2}{x}$

(e) $\lim_{x \to \infty} (\sqrt{x^2 + 3} - x)$

(f) $\lim_{x \to a} g(t)$

$$\text{where } g(t) = \begin{cases} t^2 & \text{if } x \geq 0 \\ t - 2 & \text{if } x < 0 \end{cases}$$

(9) Use the precise definition of the limit to prove the following

(a) $\lim_{x \to 4} \sqrt{x} = 2$

(b) $\lim_{x \to 5} 3x = 15$

(c) $\lim_{x \to 3} (x^2 - 5) = 4$

(d) $\lim_{x \to -3} \frac{x^2 - 9}{x + 3} = -6$